Fourier dimensionality reduction for fast radio transients

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Introduction

Fast radio transients (FRT) detection is an active field of research in radio astronomy.

- Unlike steady sources imaging, the FRT imaging suffers from the sensitivity of the integration time of the radio interferometer.
- In the SKA era, big data is a computational challenge for imaging.

Classical technique

A widely used technique to reduce the data dimensionality in the community is to average visibilities over the series of snapshots (*averaging*). However, FRT can be diluted temporally with such technique.

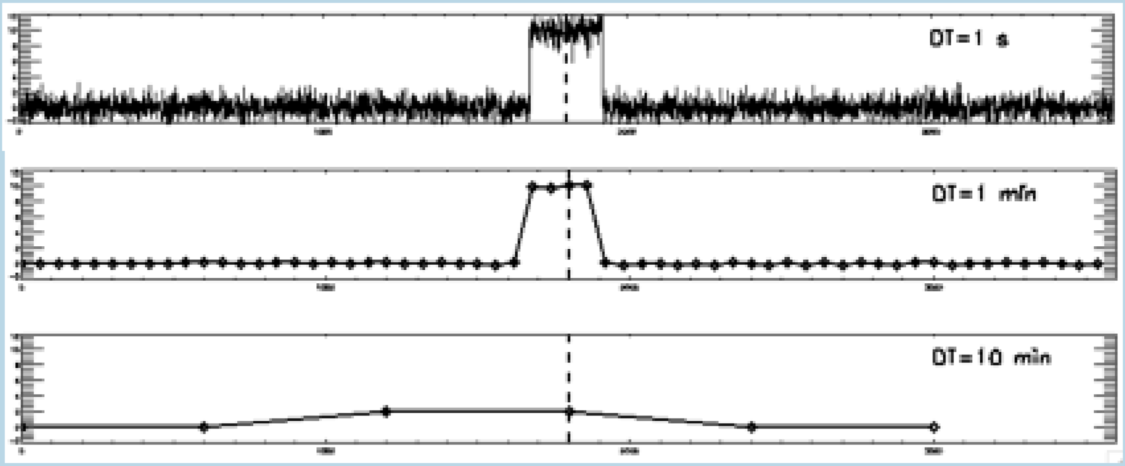


Illustration of the problem of the averaging

Fourier reduction method

The FRT imaging problem at time frame t

$$\mathbf{y}_t = \mathbf{\Phi}_t \mathbf{x}_t + \mathbf{n}_t$$

$\mathbf{y}_t \in \mathbb{C}^M$: continuous visibilities
$n \in \mathbb{C}^M$: additive i.i.d. Gaussian noise
$\mathbf{\Phi}_t \in \mathbb{C}^{M\times N}$: measurement operator
$\mathbf{x}_t \in \mathbb{R}^N$ $(M \gg N)$: sky

The $\mathbf{\Phi}_t$ is time varying in the FRT imaging, so we only spatially reduce the dimensionality.
We apply a linear operator $\mathbf{R}_t$ (at t) so as to reduce the dimensionality of the initial imaging problem such that

$$\mathbf{R}_t\mathbf{y}_t = \mathbf{R}_t\mathbf{\Phi}_t\mathbf{x}_t + \mathbf{R}_t\mathbf{n}_t.$$

Inspired by SVD decomposition, the optimal dimensionality reduction $\mathbf{R}_t$ is written as

$$\mathbf{R}_t = \mathbf{U}_{t_{th}}^{\dagger} = \mathbf{\Sigma}_{t_{th}}^{-1}\mathbf{V}_{t_{th}}^{\dagger}\mathbf{\Phi}_{t_{th}}^{\dagger},$$

where th is a threshold applied on singular values. Due to the computational demanding SVD, a fast approximation ([2]) is proposed such that ${\mathbf{V}_t}^{\dagger} \approx \mathbf{F}$ and ${\mathbf{\Sigma}_t}^2 \approx \text{Diag}(\mathbf{F}\mathbf{\Phi}_t^{\dagger}\mathbf{\Phi}_t\mathbf{F}^{\dagger})$. The final reduction operator at t is given by

$$\mathbf{R}_t = {\mathbf{\Sigma}_t}^{-1}\mathbf{S}_{th}\mathbf{F}{\mathbf{\Phi}_t}^{\dagger},$$

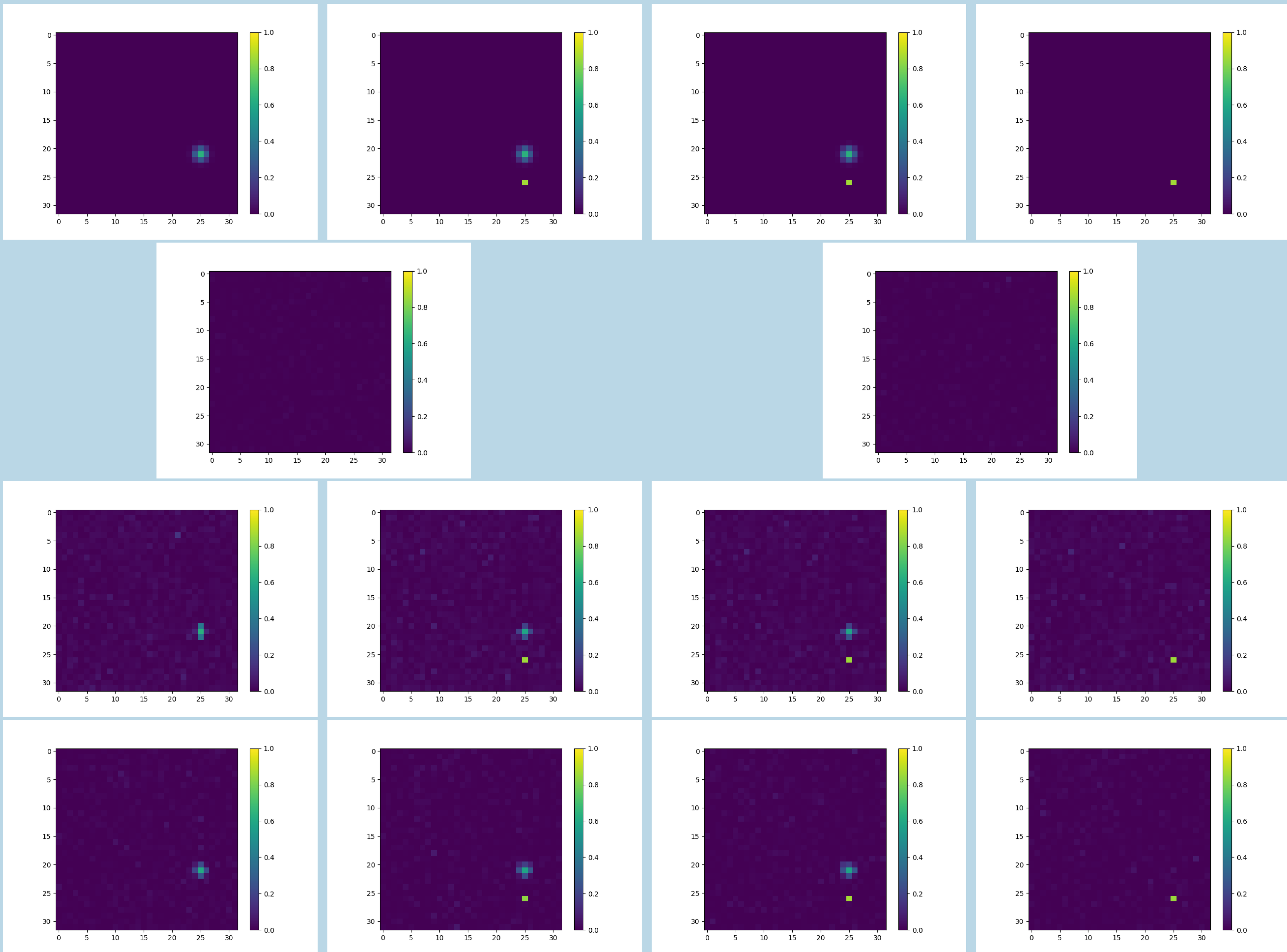
where $\mathbf{S}_{th}$ selects singular values larger than the given threshold th.
This model can be used as a pre-processing step to tackle big data challenge for the optimization-based FRT imaging methods ([1]).

Simulations

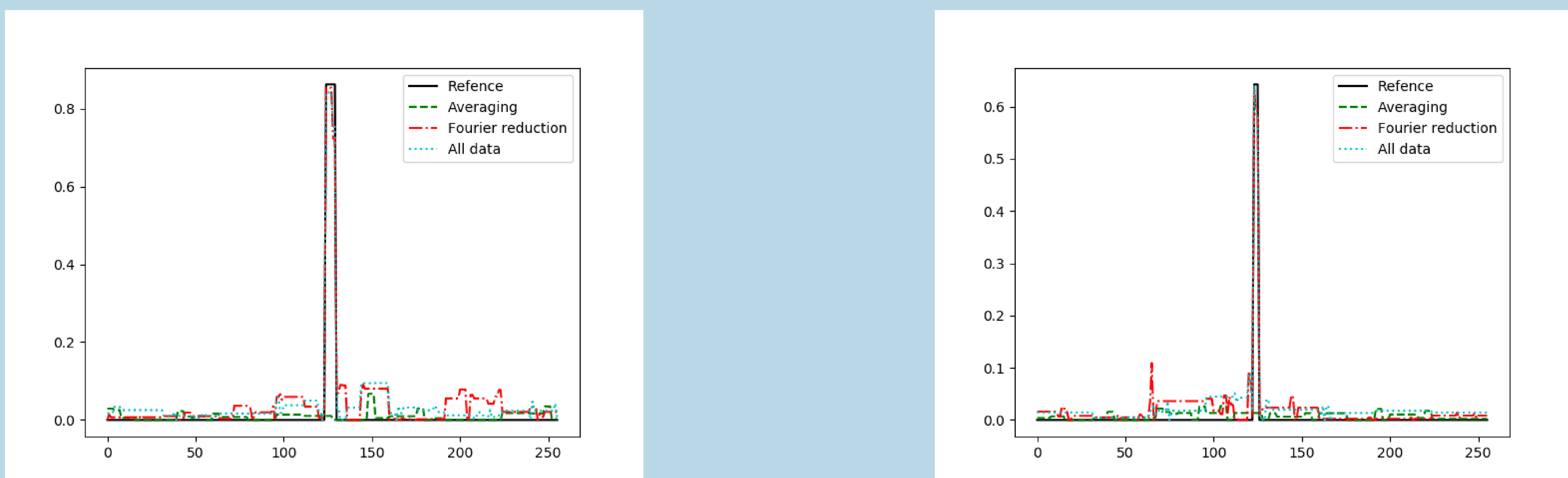
Simulation settings:

- Sky model: a cube of size $32 \times 32 \times 256$ with 256 time frames. Each time frame represents 45 seconds.
- FRT: 5 FRTs randomly appear in the sky model. Their "lifespans" are between 1.5min (2 snapshots) and 4.5min (6 snapshots).
- UV-coverage: realistic uv-coverage was simulated via MeqTree tool by using SKA 197 antennas configuration.
- Noise: Gaussian noise with SNR $= -30$dB (SNR $= 20\log_{10}(||\mathbf{y}_0||_2/||\mathbf{n}||_2)$).
- Reduction level: 25%. The temporal averaging computed the average for every 4 consecutive snapshots while our Fourier reduction method kept 25% data of each snapshot.
- Reconstruction algorithm: the FRT imaging method in [1].

Results:
The figure below compares different methods of reduction and showcases the typical problem of temporal averaging. Due to the short duration of the FRT, the temporal averaging dilutes the temporal information of FRT (second row of the figure). However, at the same level of the reduction, the proposed method does not degrade the reconstruction quality (third row of the figure), achieving almost the same result of using all of the data (fourth row of the figure).



Due to the limited space, only frames from 123 to 126 are presented. From top to bottom: sky model, reconstruction with 25% visibilities reduced by temporal averaging (the corresponding frames 31 and 32 are presented), reconstruction with only 25% visibilities reduced by the proposed Fourier reduction method, and reconstruction with all the visibilities.



With different reduction methods, the recovered temporal profiles of the two FRTs in the above figure are presented.

References

[1] M. Jiang, J. N. Girard, J. . Starck, S. Corbel, and C. Tasse: *Compressed sensing and radio interferometry*, in 2015 23rd European Signal Processing Conference (EUSIPCO), Aug 2015, pp. 1646-1650.

[2] S. Vijay Kartik, R. E. Carrillo, J.-P. Thiran, and Y. Wiaux: *A fourier dimensionality reduction model for big data interferometric imaging*, Monthly Notices of the Royal Astronomical Society, vol. 468, no. 2, pp. 2382-2400, 2017.

Acknowledgements

This research is supported by Swiss-South Africa Joint Research Program (IZLSZ2_170863 / 1). The support is gratefully acknowledged.