Joint Multichannel Deconvolution and Blind Source Separation

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Introduction

- In multichannel or multi-wavelength imaging applications, sources with different spectra are mixed together. It is of great interest to identify and extract their characteristic spectra, leading to the study of the Blind Source Separation (BSS).
- In practice, the data, such as radio interferometeric data, may be incompletely sampled or blurred due to the Point Spread Function (PSF) of the instrument. BSS becomes more challenging when dealing with such imperfect data.
- Thus, we propose our **Deconvolution Blind Source Separation (DBSS)** model which addresses joint multichannel deconvolution and BSS problems.

DBSS model

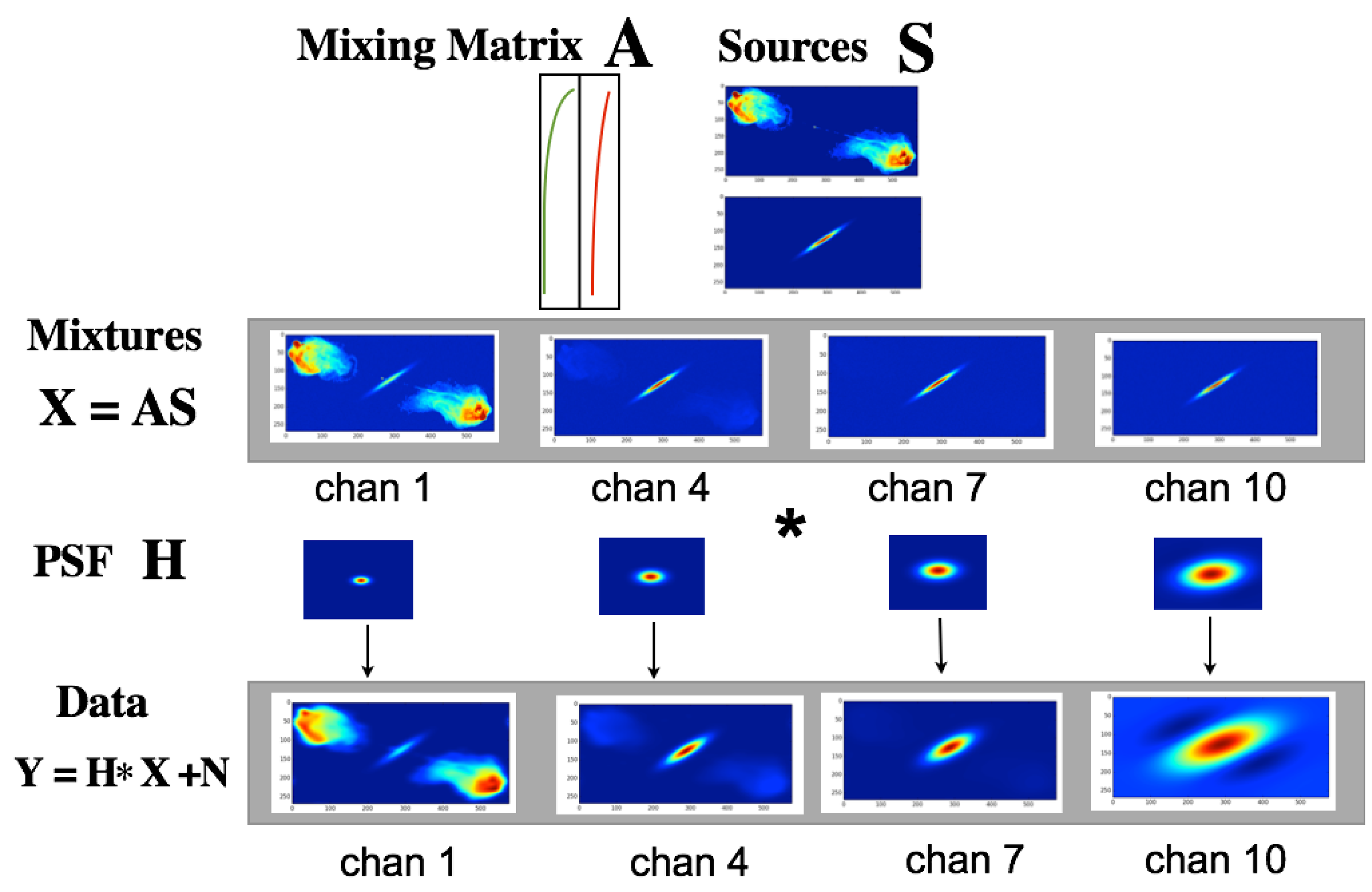


Illustration of DBSS model

Formulation

$$\forall \nu \in \{1,2,\cdots,\mathrm{N_c}\},\ \mathbf{y}_\nu = \mathbf{h}_\nu * \left(\sum_{j=1}^{\mathrm{N_s}} A_{\nu,j}\mathbf{s}_j\right) + \mathbf{n}_\nu \qquad (1)$$

Our model can be more conveniently described in the Fourier domain. In practice, imaging systems such as radio interferometry imaging and Magnetic Resonance Imaging measure Fourier components. Therefore, our model in the Fourier domain has more interests in practical applications.

Optimization problem

Prior

Sources are sparse in a known dictionary $\mathbf{\Phi}$.

Lagrangian-form minimization (corresponding to Fourier version of eq.(1))

$$\min_{\mathbf{S},\mathbf{A}} \frac{1}{2}\sum_{\nu}^{\mathrm{N_c}}\sum_{k}^{\mathrm{N_p}} ||\hat{Y}_{\nu,k} - \hat{H}_{\nu,k}\mathbf{a}_\nu\hat{\mathbf{s}}^k||_2^2 + \sum_i^{\mathrm{N_s}} \lambda_i ||\mathbf{s}_i\mathbf{\Phi}^t||_0 \qquad (2)$$

Challenges

- Non-convexity Due to the indeterminacy of the product $\mathbf{A}\hat{\mathbf{S}}$, only a critical point instead of global optimum can be expected.
- Ill-conditioning The convolution kernel $\hat{\mathbf{H}}$ can be ill-condtitioned or even rank deficient, making the deconvolution unstable.

GMCA framework

BSS solutions

- Statistical approach: ICA (FastICA), etc.
- Methods based on morphological diversity: GMCA and its variations

GMCA (Generalized Morphological Component Analysis)

- Advantage
  An efficient BSS method taking advantage of morphological diversity and sparsity in a transformed space. Compared to ICA-based methods, GMCA is more robust to noisy data.
- Limitation
  GMCA does not take deconvolution into account, which is limited in pratical applications. Therefore, a more rigorous BSS method should be conceived for the DBSS problem.

DecGMCA: A sparse DBSS method

Motivation

The original problem (2) can be split into two alternating solvable convex sub-problems: estimating $\hat{\mathbf{S}}$ and estimating $\mathbf{A}$, which can be solved by a projected alternating least-squares algorithm.

Two-stage estimation

- Estimate of $\mathbf{S}$ with respect to $\mathbf{A}$

$$\min_{\mathbf{S}} \frac{1}{2}\sum_{\nu}^{\mathrm{N_c}}\sum_{k}^{\mathrm{N_p}} ||\hat{Y}_{\nu,k} - \hat{H}_{\nu,k}\mathbf{a}_\nu\hat{\mathbf{s}}^k||_2^2 + \sum_i^{\mathrm{N_s}} \lambda_i ||\mathbf{s}_i\mathbf{\Phi}^t||_0$$

To stabilize the multichannel deconvolution step, we introduce a Tikhonov regularization of the least-square estimate in Fourier space. We can derive that:

$$\hat{\mathbf{s}}^k = \left(\sum_{\nu}^{\mathrm{N_c}} \left(\hat{H}_{\nu,k}\mathbf{a}_\nu\right)^t \left(\hat{H}_{\nu,k}\mathbf{a}_\nu\right) + \epsilon' \mathbf{I}_{\mathrm{N_s}}\right)^{-1} \sum_{\nu}^{\mathrm{N_c}} \hat{H}_{\nu,k}\hat{Y}_{\nu,k}\mathbf{a}_\nu^t$$

We then apply a sparsity prior in wavelet space and estimate the current coefficients by hard-thresholding:

$$\forall i \in \{1,2,\cdots,\mathrm{N_s}\};\quad \mathbf{s}_i = \left(\mathrm{Th}_{\lambda_i}\left(\mathbf{s}_i\mathbf{\Phi}^t\right)\right)\mathbf{\Phi}$$

- Estimate of $\mathbf{A}$ with respect to $\mathbf{S}$

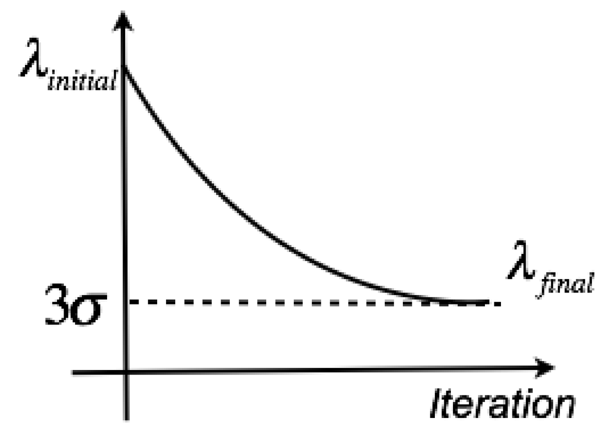
$$\min_{\mathbf{A}} \frac{1}{2}\sum_{\nu}^{\mathrm{N_c}}\sum_{k}^{\mathrm{N_p}} ||\hat{Y}_{\nu,k} - \hat{H}_{\nu,k}\mathbf{a}_\nu\hat{\mathbf{s}}^k||_2^2$$

Via least squares, the solution is:

$$\mathbf{a}_\nu = \left(\sum_{k}^{\mathrm{N_p}} \hat{H}_{\nu,k}\hat{Y}_{\nu,k}\left(\hat{\mathbf{s}}^k\right)^*\right)\left(\sum_{k}^{\mathrm{N_p}} \left(\hat{H}_{\nu,k}\hat{\mathbf{s}}^k\right)\left(\hat{H}_{\nu,k}\hat{\mathbf{s}}^k\right)^*\right)^{-1}$$

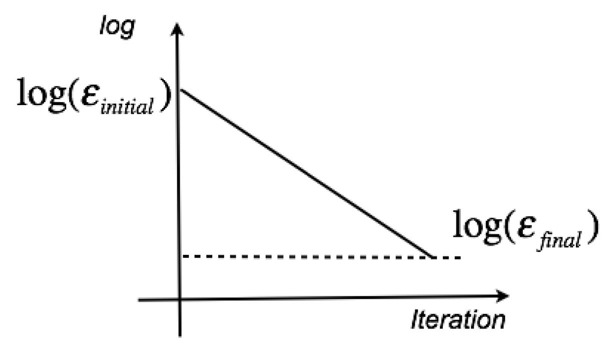
Choice of parameters

- Thresholds λ



High initial thresholds to extract the most significant features. Then, the thresholds decrease to retrieve more detailed information about the sources.

- Regularization parameter ϵ



A large ϵ makes the system more regularized, but the detailed information will be smoothed; a smaller ϵ conserves more details, but the system will be unstable.

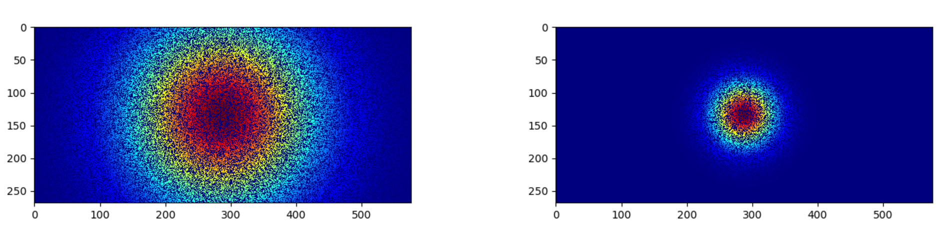
Global view of DecGMCA

1. Initialize $\mathbf{A}^{(0)}$
2. Iterate $i = 1,\cdots,\mathrm{N_i}$
   (i) Update $\mathbf{S}$ knowing $\mathbf{A}$
   (ii) Update $\mathbf{A}$ knowing $\mathbf{S}$
   (iii) Decrease the thresholding λ
   (iv) Decrease the Tikhonov parameter ϵ
3. Refinement step: with respect to $\mathbf{A}$, the estimation of the sources are ameliorated by using proximal methods

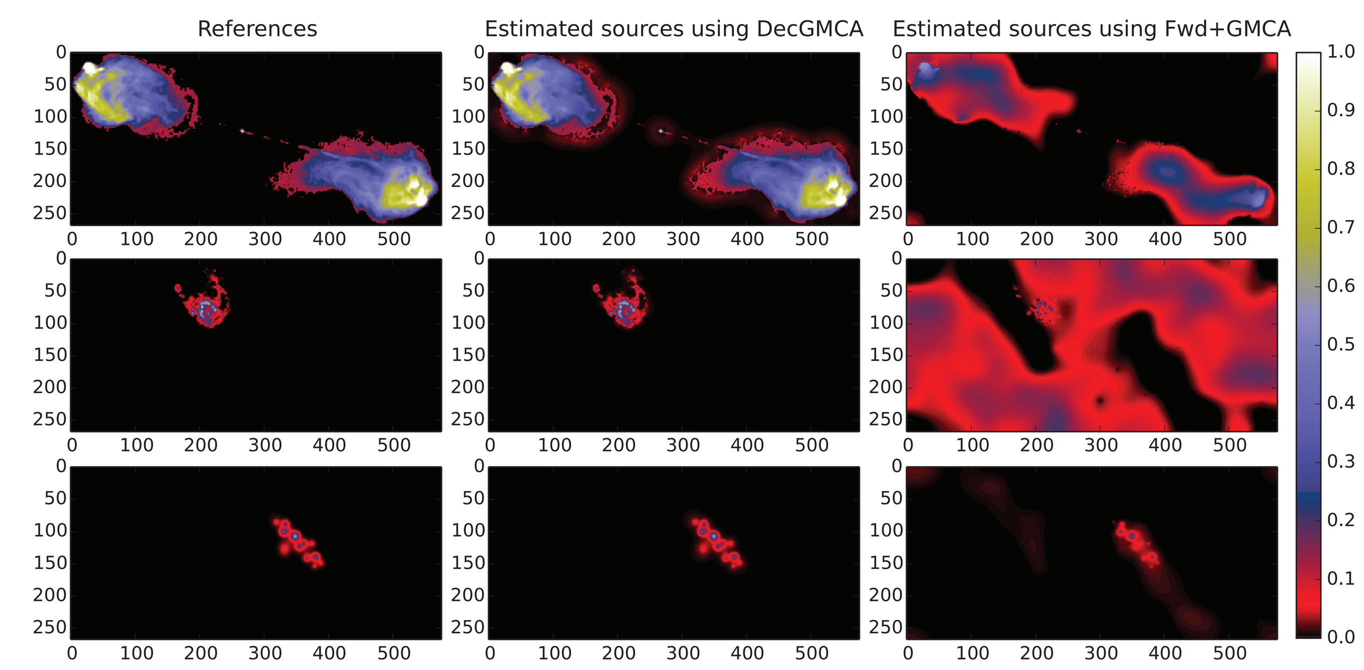
Numerical experiment

The number of observation channels is 20. The resolution of the PSF is varying along with channels. The observation is under-sampled with only 50% active data.

Example of PSF



The left figure shows the best resolved PSF in Fourier space while the right figure shows the worst resolved PSF.



Comparison with other methods

- DecGMCA: Joint deconvolution and BSS method
- ForWaRD + GMCA: Sequential deconvolution (using ForWaRD method) and BSS

We can observe that sources recovered by DecGMCA are well deconvolved and separated, which outperforms the sequential deconvolution and BSS.

Comparison of relative errors between DecGMCA and ForWaRD+GMCA

Sources	DecGMCA	ForWaRD+GMCA
1	0.14%	54.74%
2	0.27%	1279.21%
3	0.36%	30.12%

Summary

- In multichannel or multi-wavelength imaging applications, sources are mixed together and data generally have different resolutions in different channels. A rigorous BSS method should take into account the different channel resolutions, which motivates our DBSS model.
- DecGMCA is an efficient method based on a sparsity prior to solve jointly the BSS and the deconvolution problems.
- We have shown that taking into account joint BSS and deconvolution gives much better results than applying a channel by channel Deconvolution followed by a BSS.
- We look forward to applying our method on real multichannel radio interferometric data, such as LOFAR/SKA data, and compare with classical methods.

References

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