
# Joint Multichannel Deconvolution and Blind Source Separation

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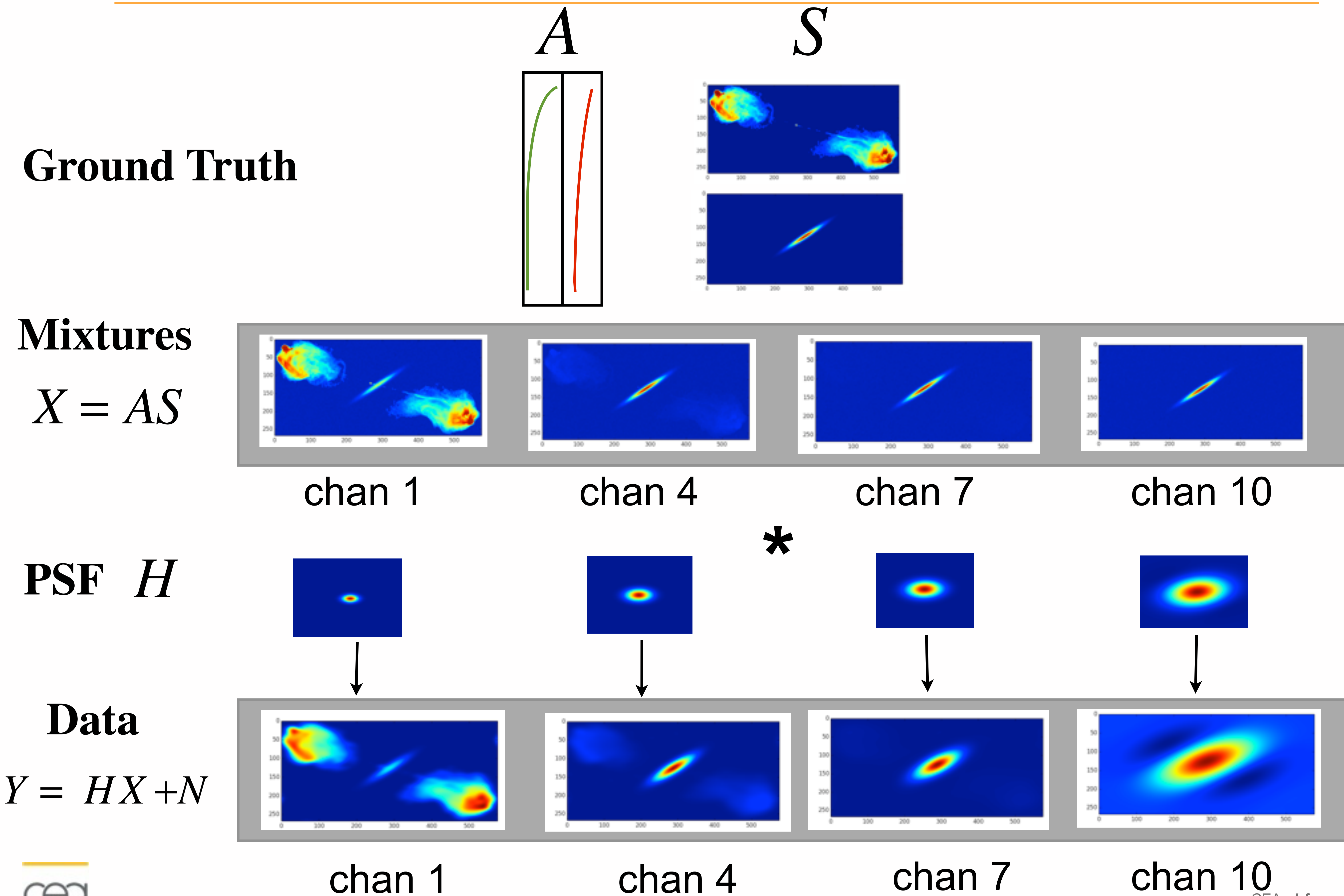
http://www.cosmostat.org/

# Outline

- **Introduction**
  - Hyperspectral data model
  - State of the art
- Joint Deconvolution and Blind Source Separation
- Experiments
- Conclusions

# Hyperspectral Data with Source Mixture Model

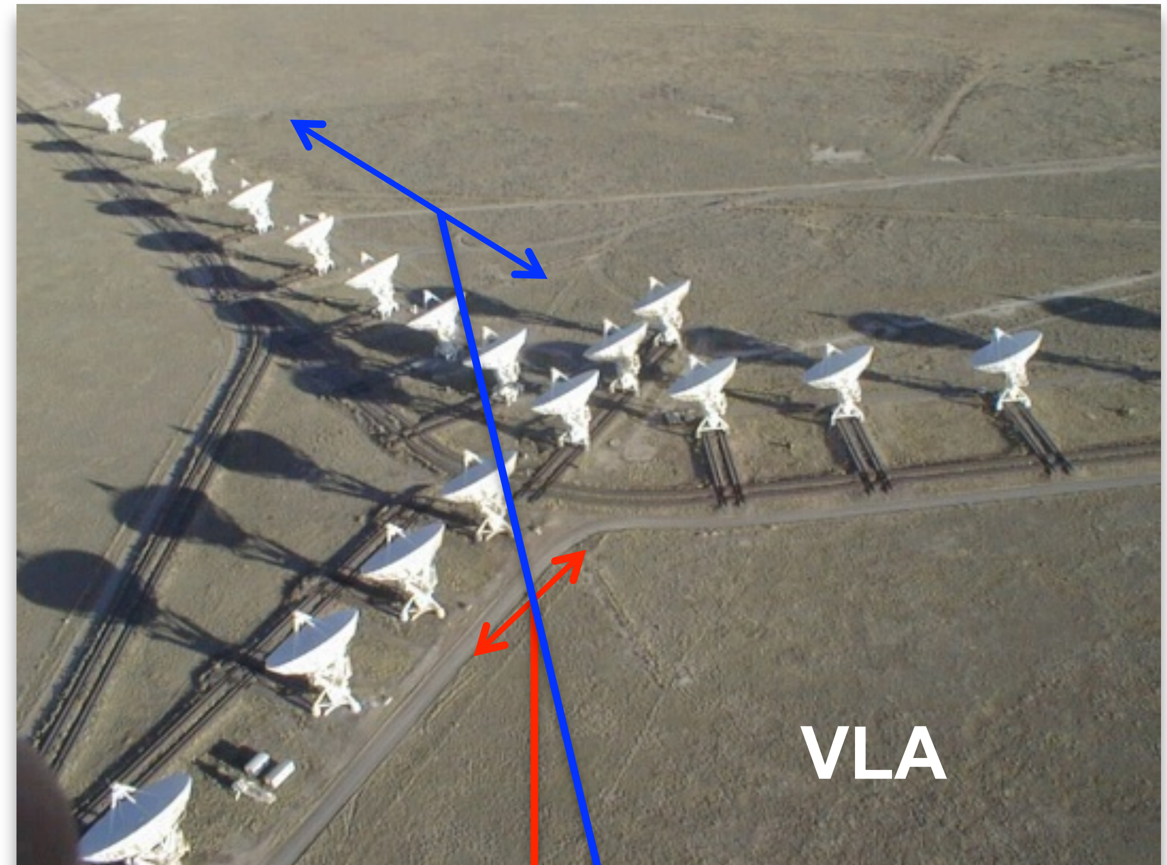


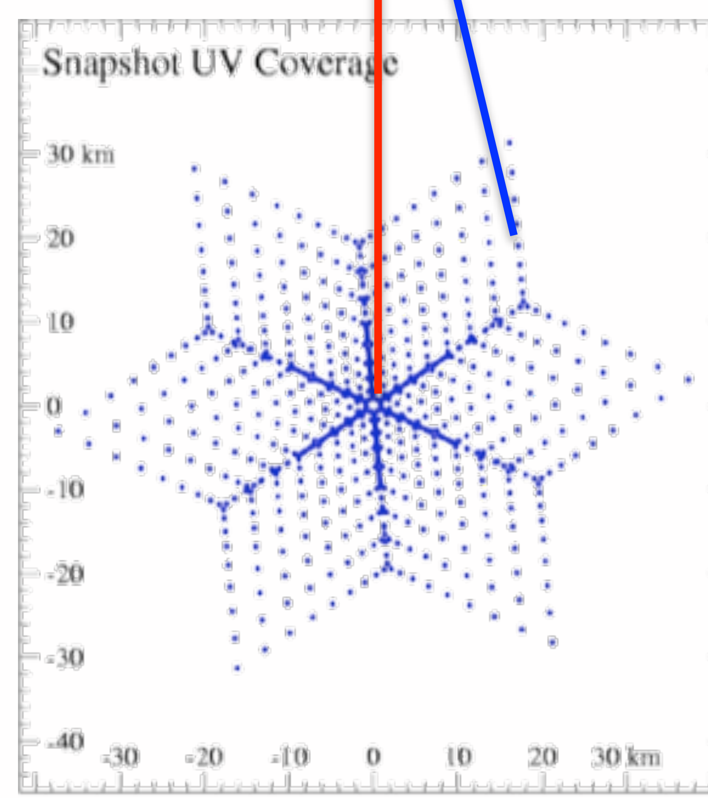
# A Specific Case: Radio Interferometry

$$Y = HX + N$$

**1 projected baseline**
**= 1 sample in the Fourier « u,v » plane**

$$V(u,v) = \int\int T(l,m) e^{-i2\pi(ul+vm)} dl dm$$




v

u

(u,v) plane sampling

## Interferometry imaging

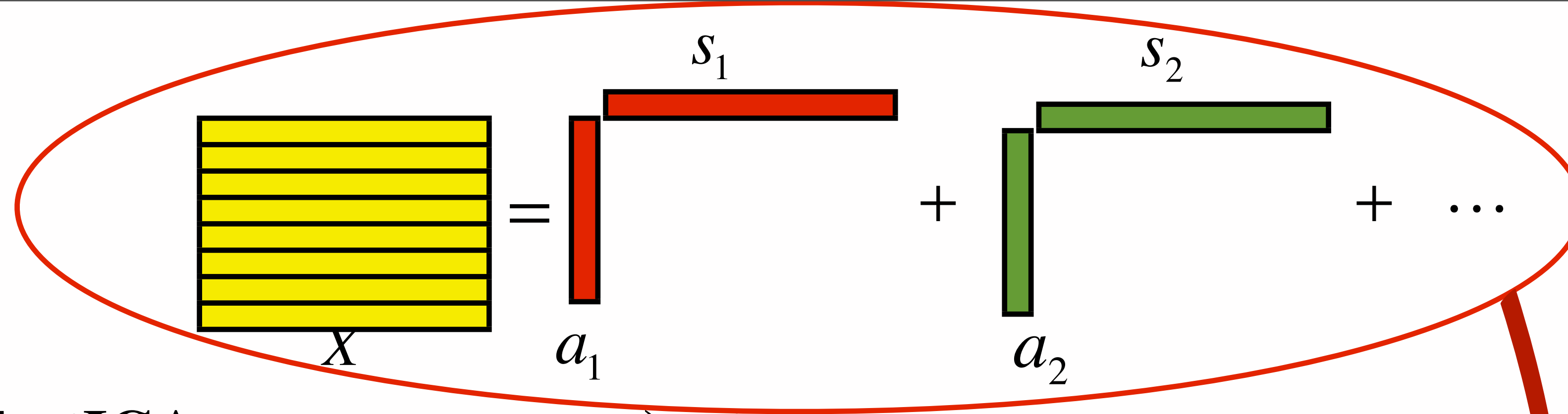
$H$ **incomplete Fourier sampling**

$$Y = HX + N = MA\hat{S} + N$$

$$\hat{S} = \mathrm{TF}(S)$$

Mask

## State of the art




- **BSS problem**

  **Statistical approach: ICA (FastICA***(A. HYVARINEN et al.)***), etc.**

  **Methods based on morphological diversity: GMCA***(J. BOBIN et al.)*** and its variations**

- **Deconvolution**

  **e.g. ForWaRD***(R.N. NEELAMANI et al.)*

$$Y = H(X) \Rightarrow X = H^{-1}(Y)$$

Joint BSS and Deconvolution?

Very few literatures!

Our method: ForWaRD+GMCA = fGMCA

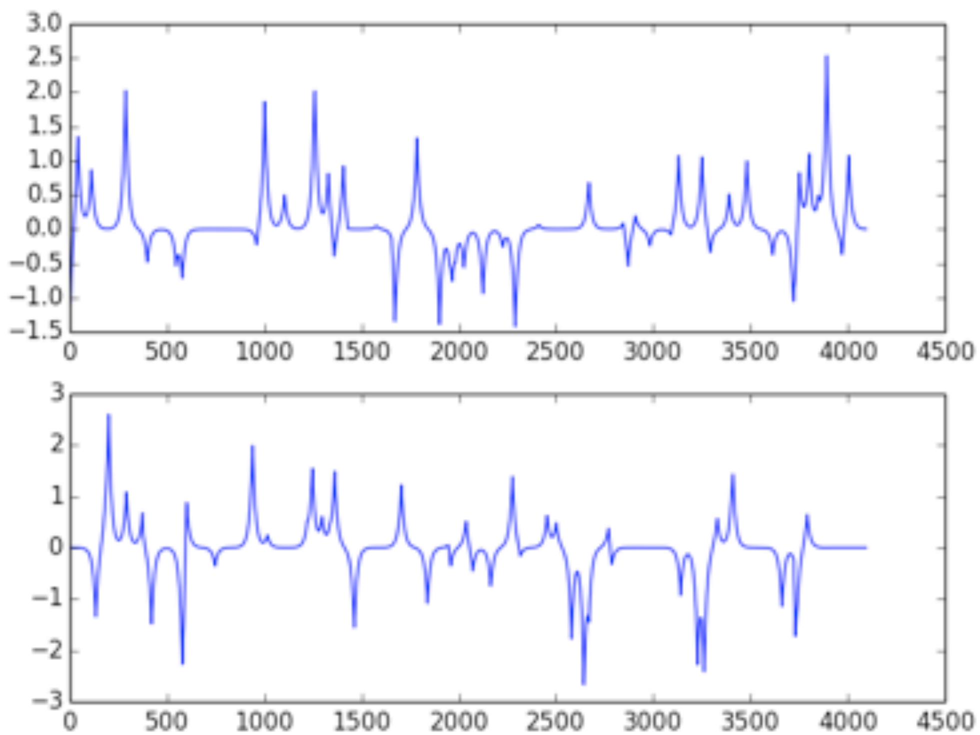
# GMCA and ForWaRD

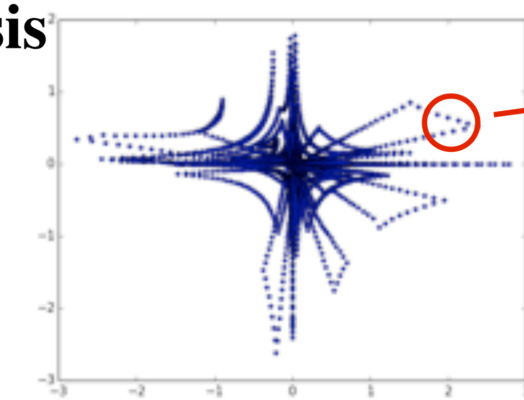
- **GMCA: Generalized Morphological Component Analysis**

1-D signals

S1

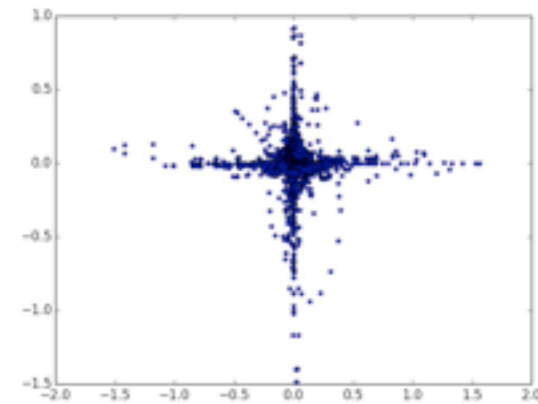
S2



Correlation

S1 v.s. S2 in direct domain

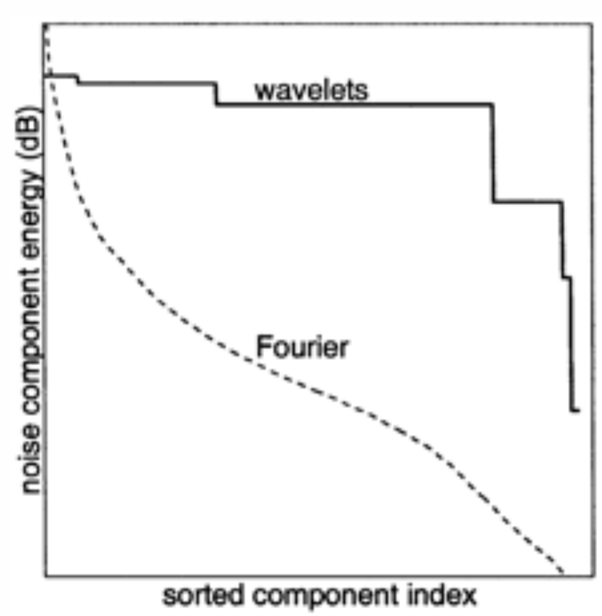


Easier to distinguish sources!

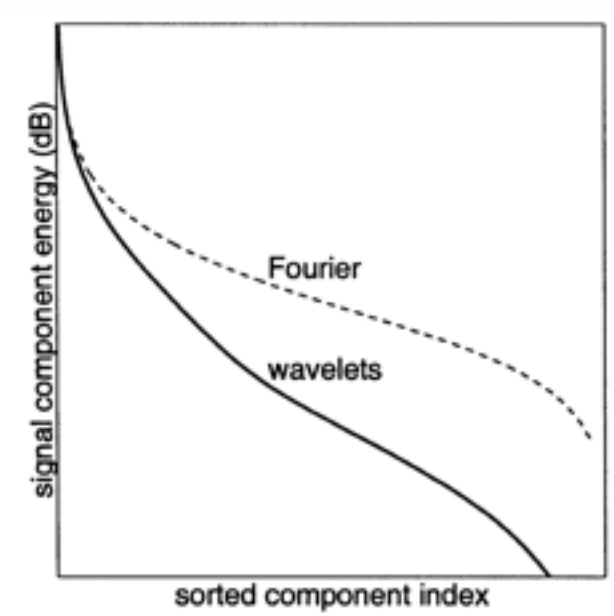
S1 v.s. S2 in wavelet domain

- **ForWaRD: Fourier-Wavelet Regularized Deconvolution**

**Fourier domain: Fourier shrinkage and regularize the ill-conditioned system**




**Wavelet domain: Wavelet shrinkage searches for smooth signals and images**

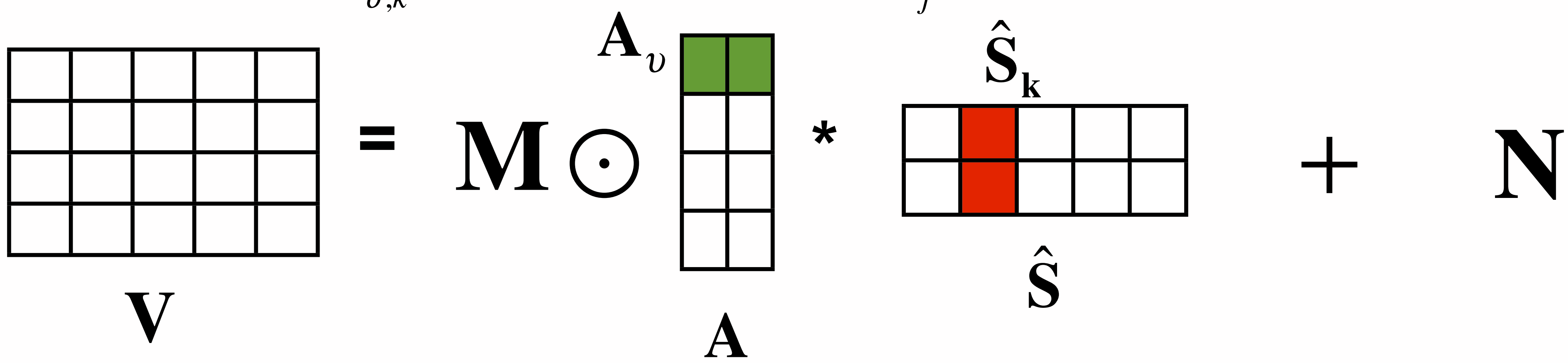



*[R.N. NEELAMANI et al.]*

# ForWaRD-GMCA(fGMCA)

- **Problem formulation**

$$\min_{\{S_{j,\bullet}\},\{A_\upsilon\}} \frac{1}{2}\sum_{\upsilon,k} \| V_{\upsilon,k} - M_{\upsilon,k}\mathbf{A}_\upsilon \hat{\mathbf{S}}_k \|_2^2 + \sum_j \lambda_j \| \mathbf{S}_{j,\bullet} \boldsymbol{\Phi}^t \|_1$$




- **Estimation of S** $\min_{\{S_{j,\bullet}\}} \frac{1}{2}\sum_{\upsilon,k} \| V_{\upsilon,k} - M_{\upsilon,k}\mathbf{A}_\upsilon \hat{\mathbf{S}}_k \|_2^2 + \sum_j \lambda_j \| \mathbf{S}_{j,\bullet} \boldsymbol{\Phi}^t \|_1$

$$\hat{\mathbf{S}}_k = \left(\sum_\upsilon (M_{\upsilon,k}\mathbf{A}_\upsilon)^t (M_{\upsilon,k}\mathbf{A}_\upsilon) + \varepsilon' \mathbf{I_n}\right)^{-1} \sum_\upsilon M_{\upsilon,k} V_{\upsilon,k} \mathbf{A}_\upsilon^t$$

$$\mathbf{S}_{wt} = \mathbf{S}\ \Phi^t$$

$$\mathbf{S}_{wt} = \mathrm{HT}_\lambda(\mathbf{S}_{wt})$$

Sparsity priori in wavelet domain

Tikhonov parameter to avoid non inversion

- **Estimation of A** $\min_{\{A_\upsilon\}} \frac{1}{2}\sum_{\upsilon,k} \| V_{\upsilon,k} - M_{\upsilon,k}\mathbf{A}_\upsilon \hat{\mathbf{S}}_k \|_2^2$

$$\mathbf{A}_\upsilon = \left(\sum_k M_{\upsilon,k} V_{\upsilon,k} \hat{\mathbf{S}}_k^*\right)\left(\sum_k (M_{\upsilon,k}\hat{\mathbf{S}}_k)(M_{\upsilon,k}\hat{\mathbf{S}}_k)^*\right)^{-1}$$

normalize the columns of **A**

- **ForWaRD-GMCA algorithm**

  - Initialize $A^{(0)}$
  - Iterate i=1,...,Niter
    - Update S knowing A $\min_{\{S_{j,\cdot}\}} \frac{1}{2} \sum_{\upsilon,k} \| V_{\upsilon,k} - M_{\upsilon,k} \mathbf{A}_\upsilon \hat{\mathbf{S}}_k \|_2^2 + \sum_j \lambda_j \| \mathbf{S}_{j,\cdot} \mathbf{\Phi}^t \|_1$
    - Update A knowing S $\min_{\{A_\upsilon\}} \frac{1}{2} \sum_{\upsilon,k} \| V_{\upsilon,k} - M_{\upsilon,k} \mathbf{A}_\upsilon \hat{\mathbf{S}}_k \|_2^2$
    - Decrease the thresholding $\lambda$ (next slide)
    - Decrease the Tikhonov parameter $\varepsilon$ (next slide)

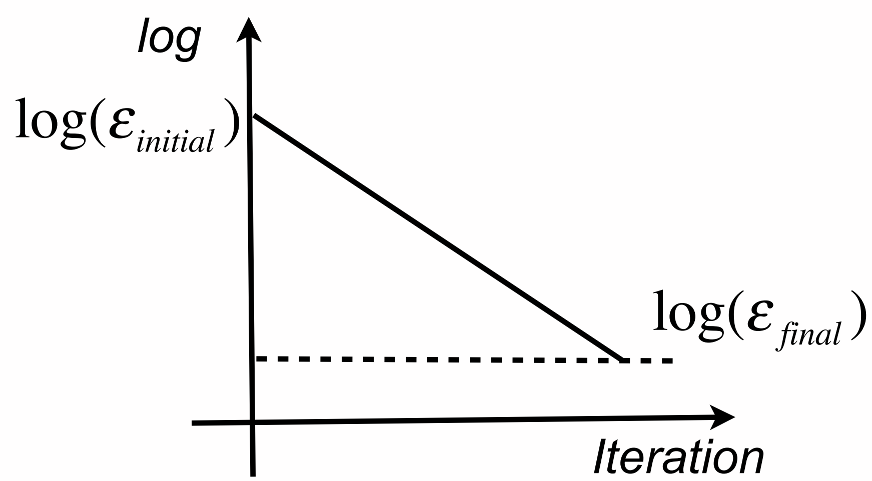
- **Choice of $\varepsilon'$**

$$\hat{\mathbf{S}}_k = \left(\sum_{\upsilon}(M_{\upsilon,k}\mathbf{A}_{\upsilon})^t(M_{\upsilon,k}\mathbf{A}_{\upsilon}) + \varepsilon'\mathbf{I_n}\right)^{-1}\sum_{\upsilon} M_{\upsilon,k}V_{\upsilon,k}\mathbf{A}_{\upsilon}^t$$

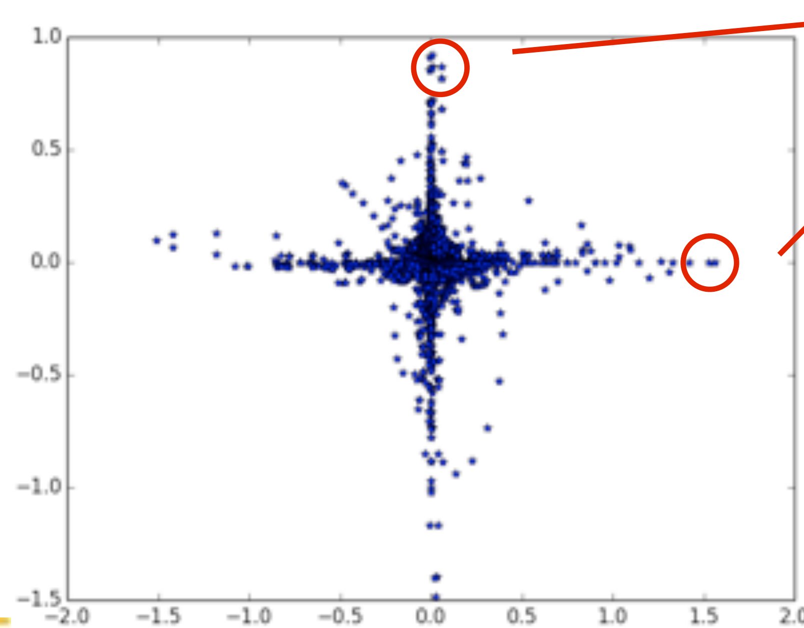
different frequency index *k*, the condition number is different.

$$\varepsilon' = \varepsilon\,\mathrm{SP}\left(\sum_{\upsilon}(M_{\upsilon,k}\mathbf{A}_{\upsilon})^t(M_{\upsilon,k}\mathbf{A}_{\upsilon})\right)$$

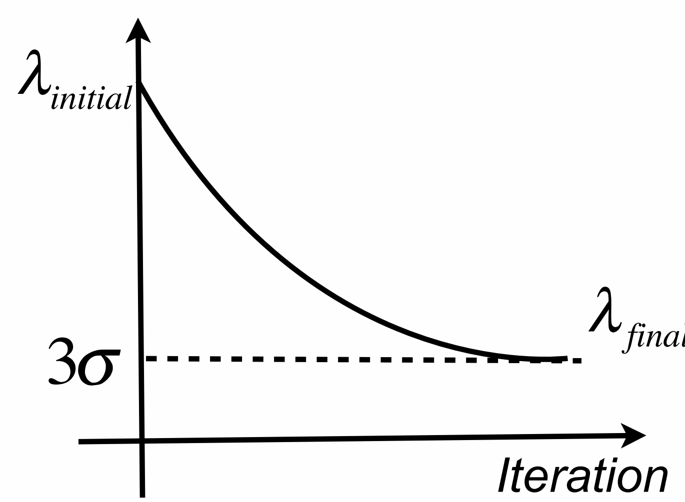
SP(**M**) is spectral radius of matrix **M**



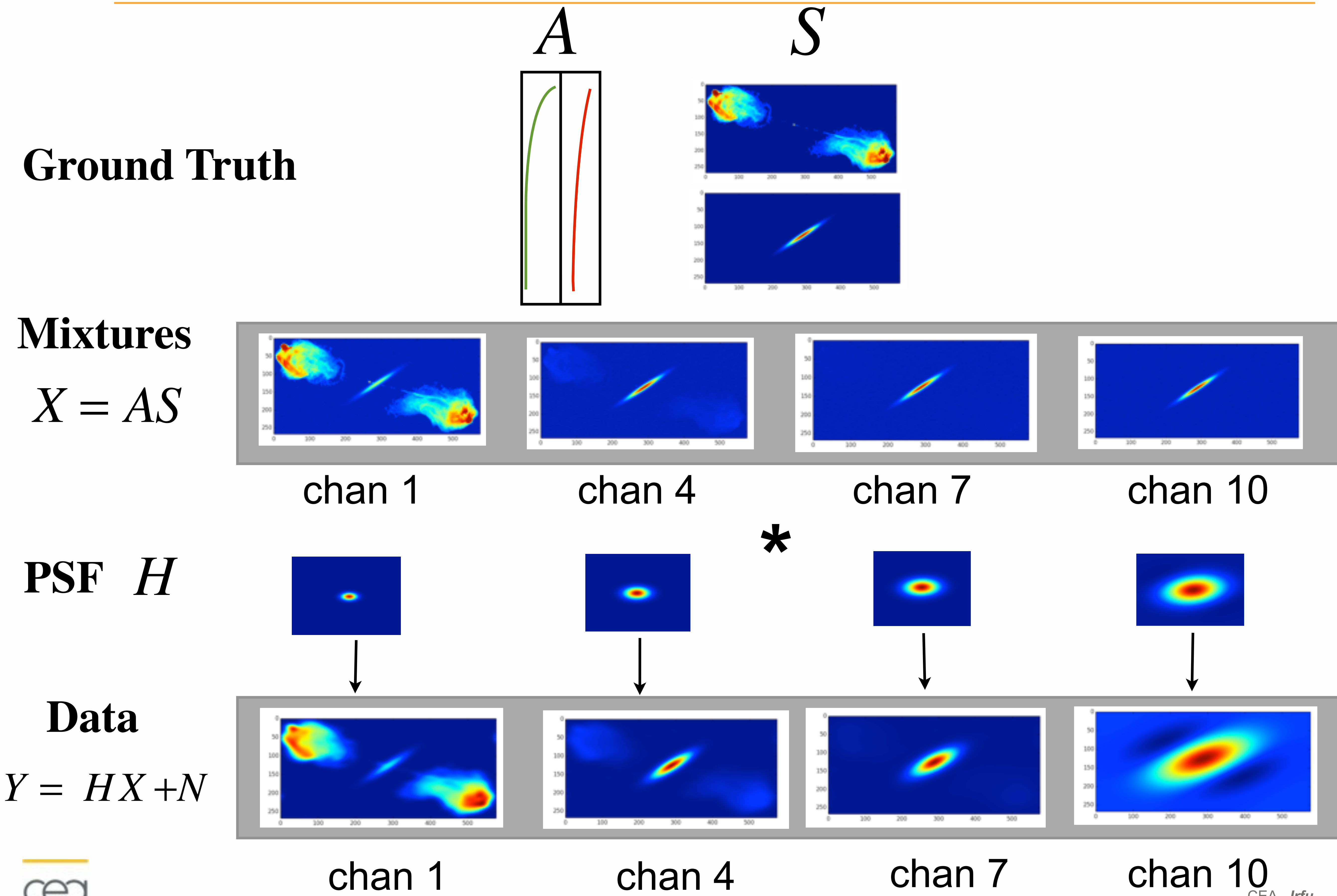

- **Choice of $\lambda$**




Larger initial threshold to select significant feature, easier to separate sources

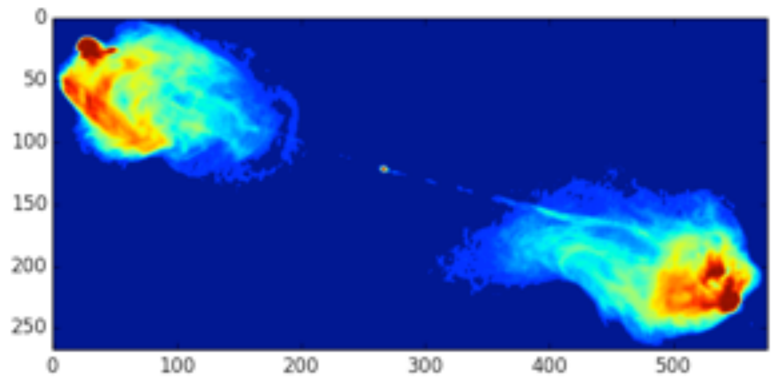


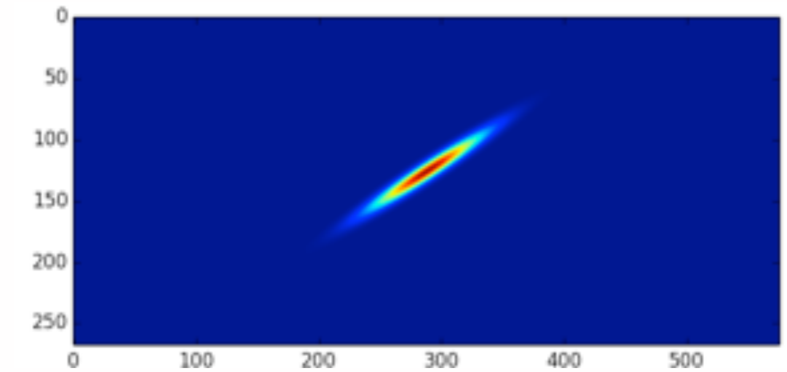
# Experiments



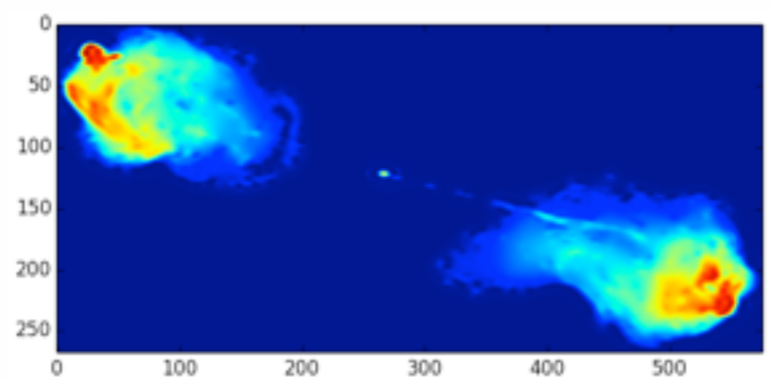
# Experiments(Source reconstruction)

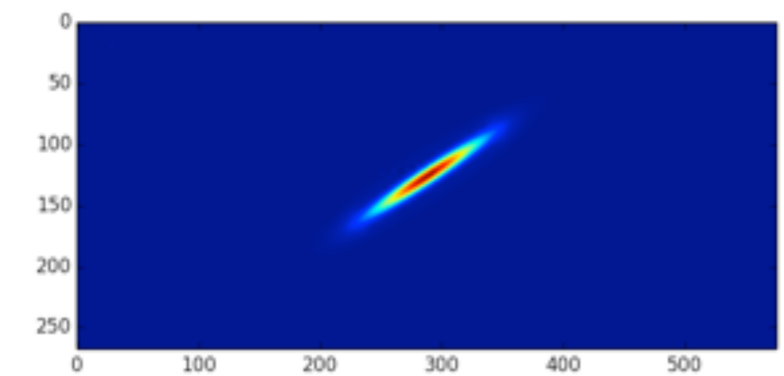
**Reference**



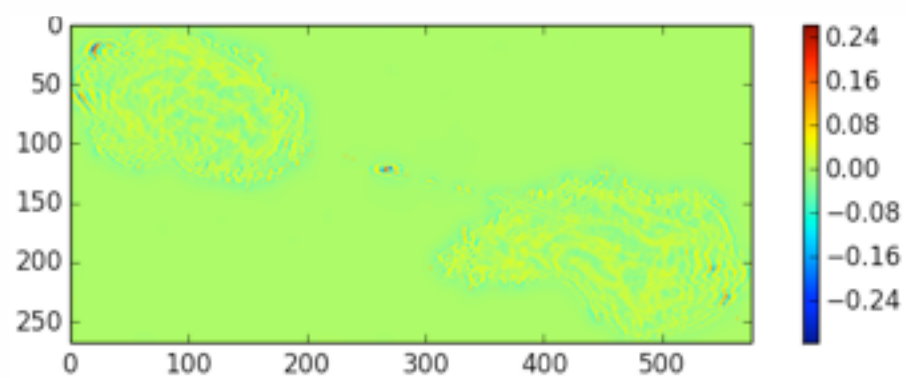



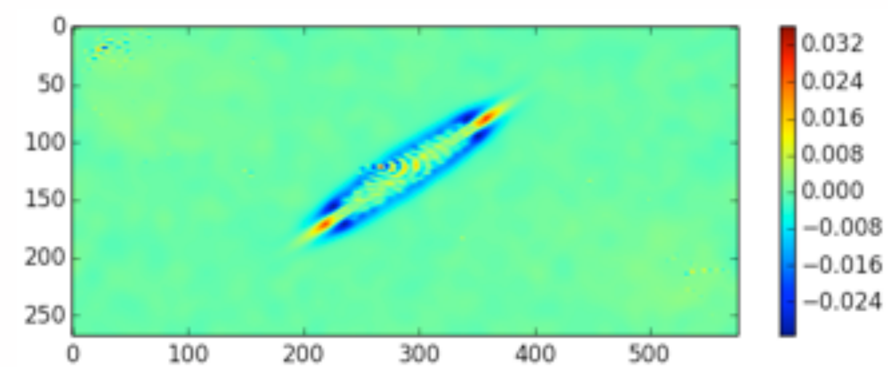

**Reconstruction**




**Error**



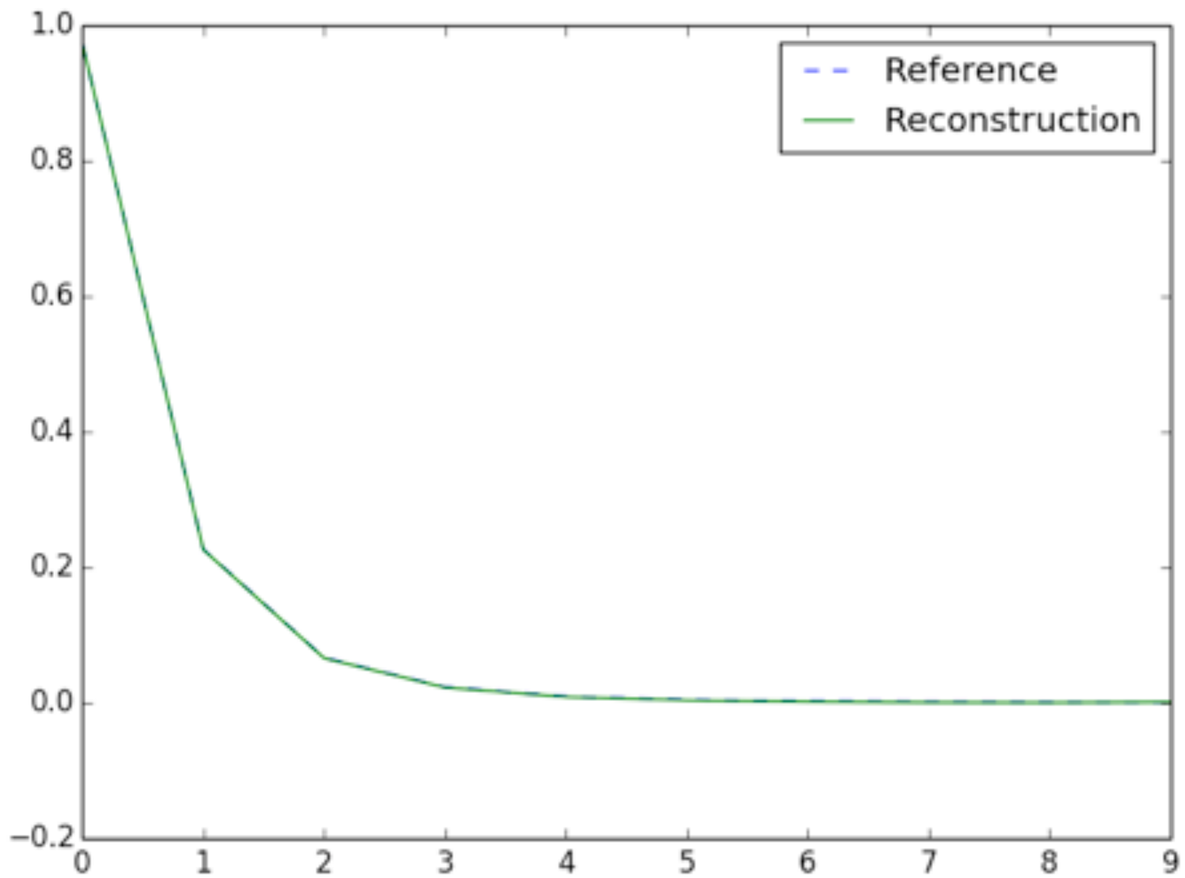




**Relative error**

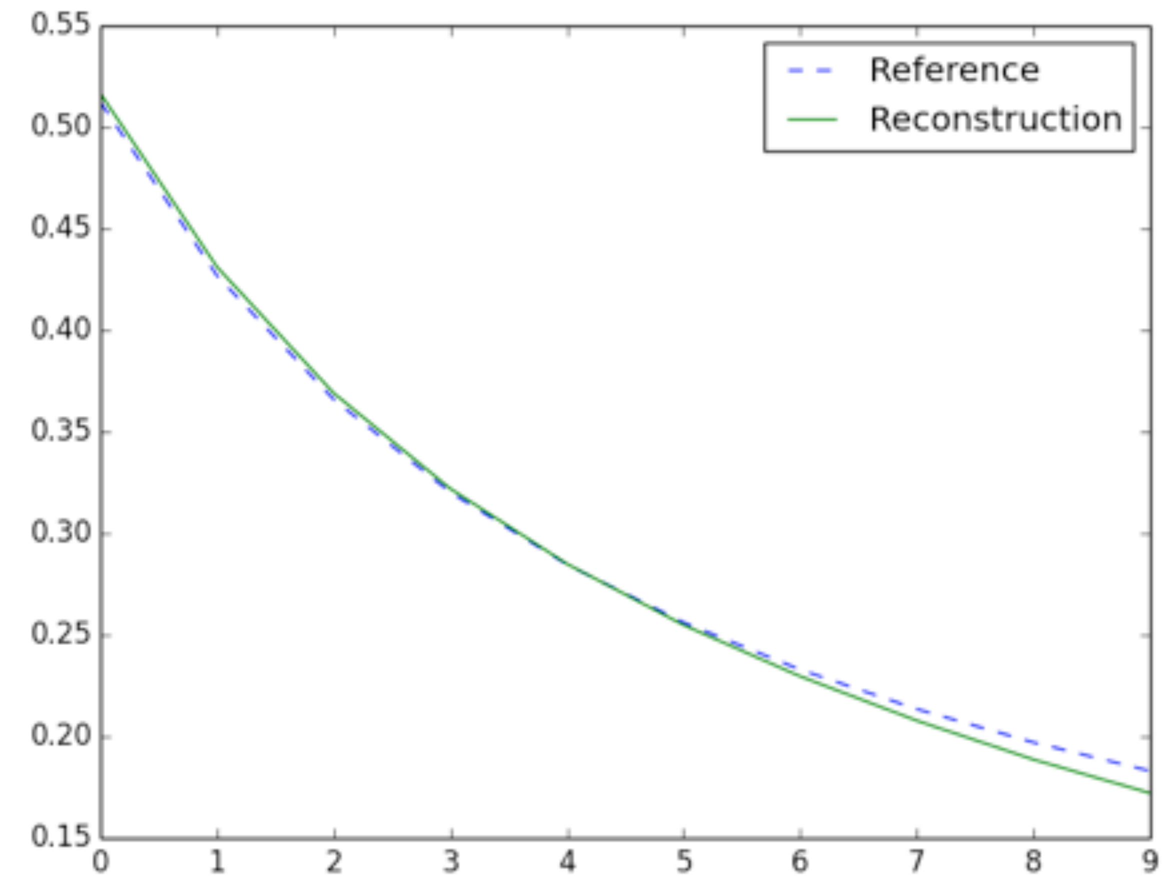
0.42%

0.19%

# Experiments(Spectra reconstruction)



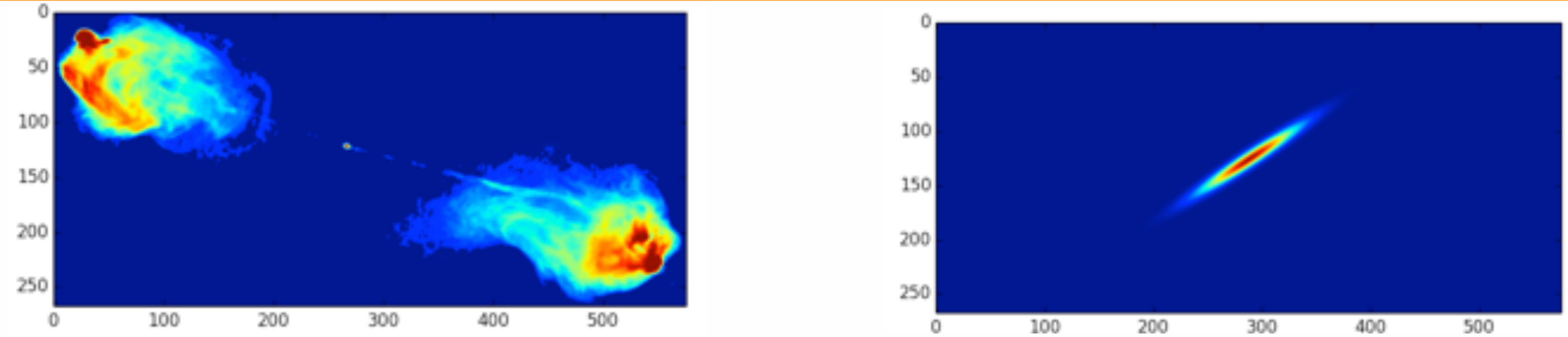

Reconstructed spectrum of $S_0$ v.s reference



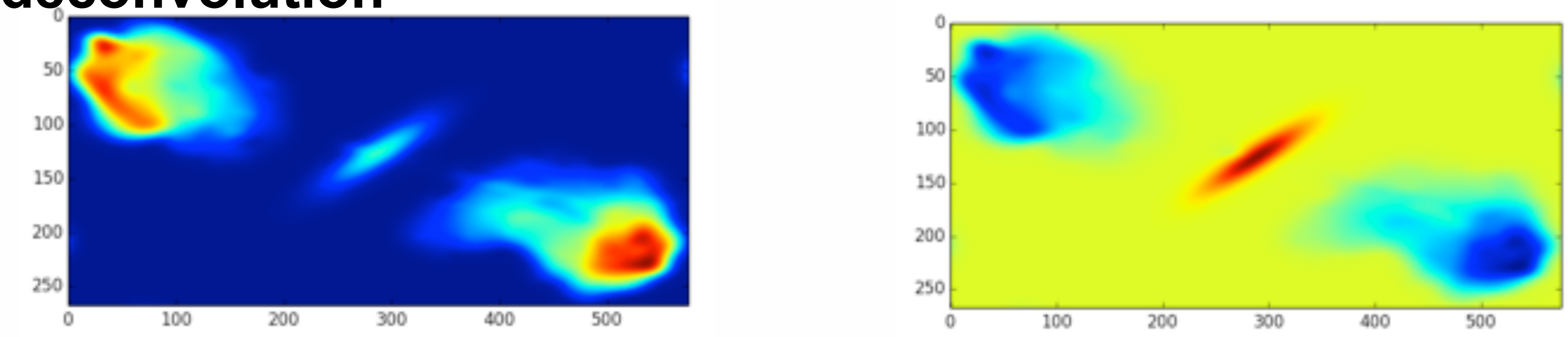

Reconstructed spectrum of $S_1$ v.s reference

# Experiments(Source reconstruction)

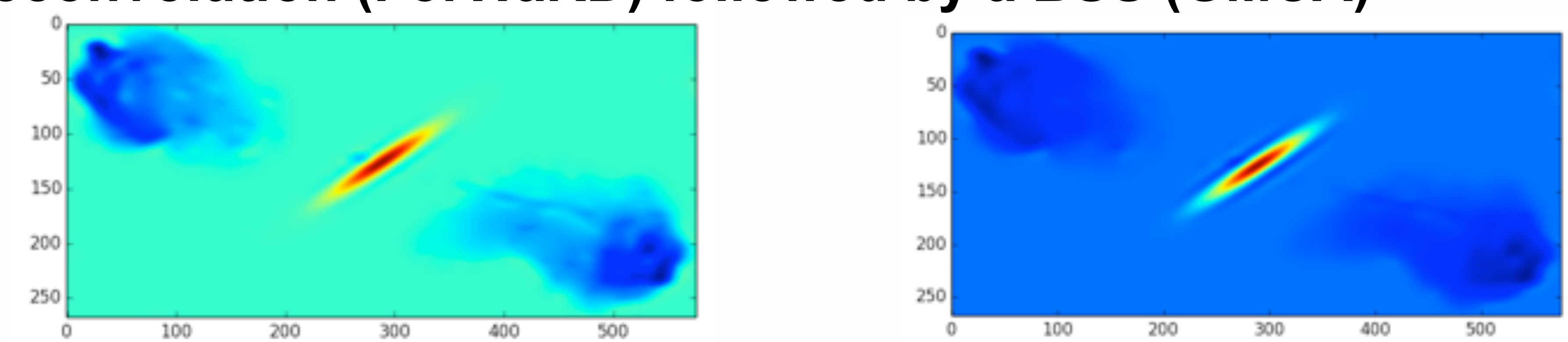
**Model sources**



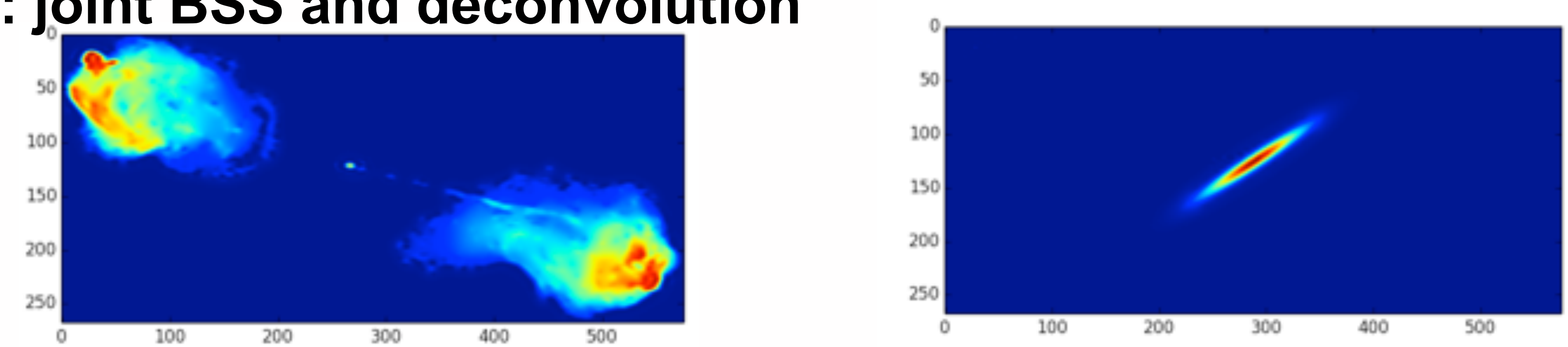

**BSS only (GMCA), no deconvolution**




**Channel by channel deconvolution (ForWaRD) followed by a BSS (GMCA)**




**Our method fGMCA : joint BSS and deconvolution**



# Conclusions and perspectives

- **Multi or hyperspectral data generally present channels at different resolution. A rigorous Blind Source Separation method should take into account the different channel resolutions.**

- **fGMCA is an efficient method to solve jointly the BSS and the deconvolution problems.**

- **It is shown that taking into account joint BSS and deconvolution gives much better results than applying only a BSS or a channel per channel Deconvolution followed by a BSS.**

- **Application on radio images(LOFAR, SKA)**

# Thank you!