Multi-channel Compressed Sensing and Radioastronomy

Radio Interferometric Sources Detection and Hyperspectral Restoration

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Introduction and Context

- In radioastronomy, the study of known class of transient sources and the recent discovery of new class of transients has motivated further development for transient detection and characterization
- Due to the limited number of baselines, the measurements (visibilities) obtained by interferometric imaging are incomplete on the Fourier plane. It's related to a deconvolution problem : Classical methods such as CLEAN and its derivates or modern method such as sparse image reconstruction in the CS framework.
- Radio sources detection problems: On the one hand, short time integration enables temporal monitoring of a transient, but each snapshot provides poor visibility coverage. On the other hand, long time integration ensures a good sampling, but it will dilute the transient.

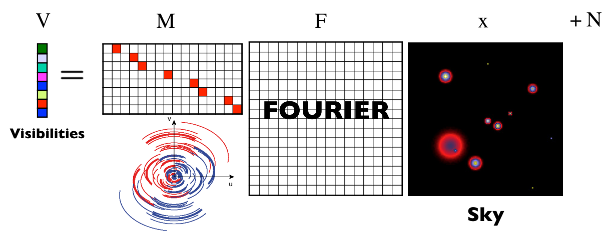
It is difficult to use classical imagers to detect and image transient source when the temporal variability of the transient source is unknown. There is an interest in developing fast imagers, enable to cope with the time variability of the sources. Such imagers can rely on the CS framework to give a quick approximate of the true sky, giving access to faster transients. This motivated the development of a 2D-1D sparse reconstruction imager.

Problem formulation

The interferometry imaging problem constitutes an ill-posed inpainting problem which can be described mathematically in Eq. (??):

$$\mathbf{V} = \mathbf{MFx} + \mathbf{N} \quad (1)$$

V:measured visibility vector,
M:sampling mask which accounts for incomplete sampling in the Fourier space,
F:Fourier Transform operator (related to interferometric imaging),
x:sky,
N:the noise (complex value).



2D1D sparse reconstruction

In analysis framework, the Eq. **??** can be converted to the following convex optimization problem:

$$\min_{\mathbf{x}} ||\mathbf{V} - \mathbf{MFx}||_2^2 + ||\mathbf{W} \odot \boldsymbol{\lambda} \odot \boldsymbol{\Phi}^t \mathbf{x}||_1 + i_{\mathbb{C}^+}(\mathbf{x}), \quad (2)$$

- **The 2D1D dictionary Φ**:
  $\psi(x,y,t) = \psi^{(x,y)}(x,y)\psi^{(t)}(t)$
  $\psi^{(x,y)}$: Starlet Transform
  $\psi^{(t)}$: Decimated transforms (e.g. CDF 9/7) depending on the temporal profile of the transient.
- **Reweighted ℓ_1 scheme**:
  After a biased solution **x** obtained by the non-reweighted convex optimization, a weighting step is performed using the following weighting strategy:

$$w_{i,j} = f(|\alpha_{i,j}|) = \begin{cases} \frac{k\sigma_j}{|\alpha_{i,j}|} & \text{if } |\alpha_{i,j}| \geq \epsilon, \\ 1 & \text{else} , \end{cases} \quad (3)$$

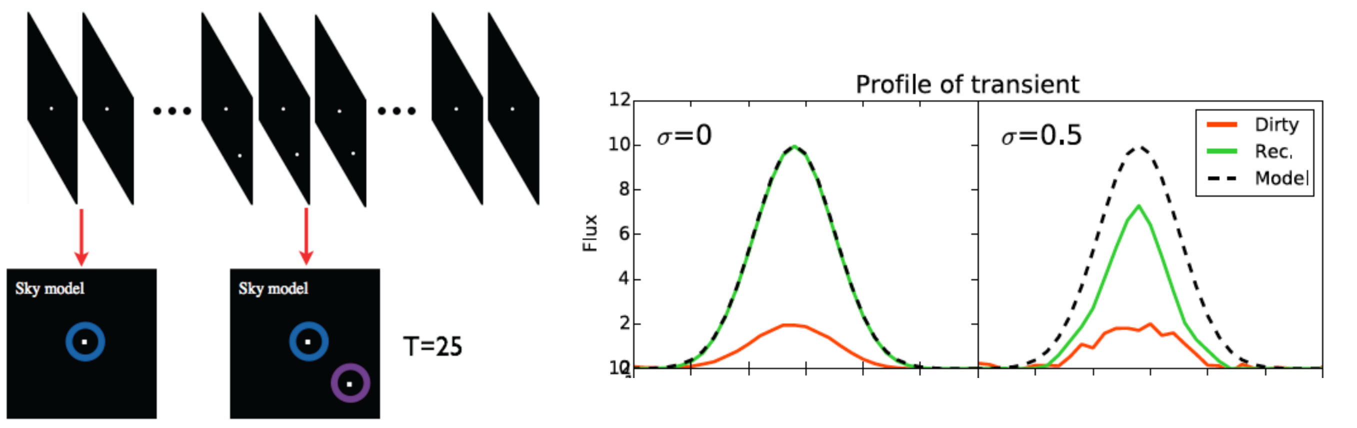
- **The whole algorithm using Condat-Vũ Splitting method**

Algorithm 1: Analysis reconstruction using CVSM
Data: Visibility **V**; Mask **M**
Result: Reconstructed image **x**

1. Initialize $(\mathbf{x}^{(0)}, \mathbf{u}^{(0)}), \mathbf{W}^{(0)} = \mathbf{1}, \tau > 0, \eta > 0, \mu \in]0,1]$;
2. **for** $n = 0$ **to** $N_{max} - 1$ **do**
3. $\quad \mathbf{p}^{(n+1)} = \mathrm{Proj}_{\mathbb{C}^+}(\mathbf{x}^{(n)} - \tau\Phi\mathbf{u}^{(n)} + \tau(\mathbf{MF})^*(\mathbf{V} - \mathbf{MFx}^{(n)}))$;
4. $\quad \mathbf{q}^{(n+1)} = (\mathbf{Id} - \mathrm{ST}_{\boldsymbol{\lambda}\odot\mathbf{W}})(\mathbf{u}^{(n)} + \eta\Phi^T(2\mathbf{p}^{(n+1)} - \mathbf{x}^{(n)}))$;
5. $\quad (\mathbf{x}^{(n+1)}, \mathbf{u}^{(n+1)}) = \mu(\mathbf{p}^{(n+1)}, \mathbf{q}^{(n+1)}) + (1-\mu)(\mathbf{x}^{(n)}, \mathbf{u}^{(n)})$;
6. $\quad \boldsymbol{\alpha}^{(n)} = \boldsymbol{\Phi}^t \mathbf{x}^{(n)}$;
7. $\quad$ Update **W** by $w_{i,j}^{(n+1)} = f(|\alpha_{i,j}^{(n)}|)$;
8. **end**
9. **return** $\mathbf{x}^{(N_{\max})}$

Results and Conclusions

- **Sky Model**:
  Cube size: 32×32 pixels on image plane and 64 frames on time. It is constituted of a control steady source at the center of the field and a transient source with a gaussian light curve. Both sources have the same peak flux density of 10 arbitrary unit in the sky model. The left figure below illustrates the sky model.

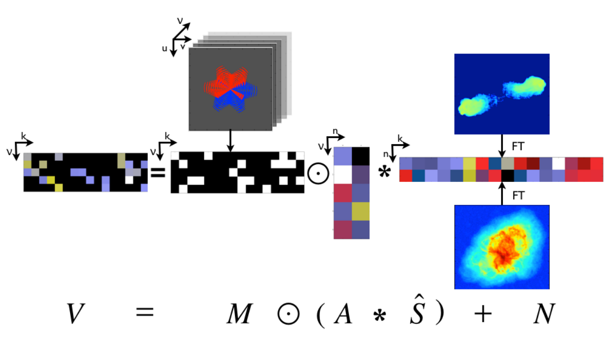


- **Reconstruction (with different noise levels)** The right figure shows time profiles at the spatial location of the transient source from the sky model, the dirty cube and the reconstructed cube for various levels of additive Gaussian noise.

  From above, we can conclude that with no additional noise (but with the sampling noise due to missing data), our 2D-1D CS method gives a perfect profile reconstruction, with flux unit relative error $\sim 10^{-5}$. As the noise level increases, the flux density of the transient is more spread around the central pixel, resulting in a bias of the peak. However, by summing the flux of the transient on nearby pixels (over a surface equal to the source size), the profile of the transient is again well recovered.

Introduction of multi-channel data

- The hyperspectral restoration becomes challenging while taking into account not only the spectral mixing, but also the deconvolution issue(e.g. PSF, incomplete data) because of instrument limit.



A: mixing matrix
Ŝ: Fourier transfor of sources

- Having defined A_ν as row vector and $\hat{S}_k$ as column vector, the optimization problem is presented as:

$$\min_{\{S_{j,\cdot}\},\{A_\nu\}} \frac{1}{2}\sum_{\nu,k} ||v_{\nu,k} - m_{\nu,k}A_\nu\hat{S}_k||_2^2 + \sum_j \lambda_j ||S_{j,\cdot}\Phi^t||_1 \quad (4)$$

HSS-ForWaRD

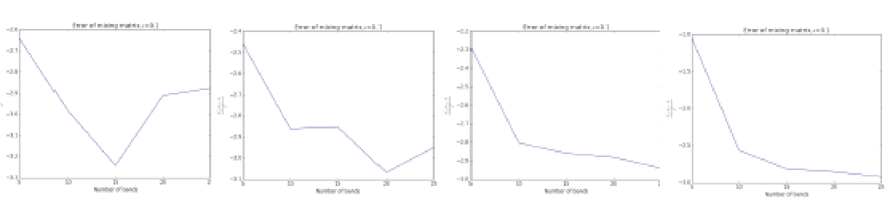
We propose a new hyperspectral sources separation inverse problem method on Fourier domain based on prior sparsity knowledge in direct or wavelet domain. Our method updates alternatively the evaluation of mixing matrix and sources while integrating the deconvolution procedure and noise regularization.

Algorithm 2: HSS-ForWaRD
Input: Visibility V; Mask M; Maximum Iterations I_{max}; Final thresholding level τ^∞
Output: Sources S; Mixing Matrix A

1. Initialization $A^{(0)} = randn(B,n)$,$S^{(0)} = randn(n,P)$,$\tau^{(0)} = 7 \sim 10$;
2. **for** $i = 1$ **to** I_{max} **do**
3. $\quad$ Estimate S knowing A: $\hat{S}_k = \mathrm{ST}_\lambda(\sum_\nu m_{\nu,k}A_\nu^* A_\nu + \epsilon I_n)^{-1}\sum_\nu m_{\nu,k}v_{\nu,k}A_\nu^*$;
4. $\quad S^{(i)} = \Re(\mathrm{TF}^{-1}(\hat{S}^{(i)}))$ `/* Real part of sources */`;
5. $\quad$ Prior sparsity knowledge:$S^{(i)} = \mathrm{ST}_\lambda(S^{(i)})$ `/*` $\lambda = \tau^{(i-1)} * \mathrm{MAD}(S^{(i)})$ `*/`;
6. $\quad$ Estimate A knowing $\hat{S}$: $A_\nu = (\sum_k m_{\nu,k}v_{\nu,k}\hat{S}_k^*)(\sum_k m_{\nu,k}\hat{S}_k\hat{S}_k^*)^{-1}$;
7. $\quad A^{(i)} = \Re(A^{(i)})$;
8. $\quad$ *Normalization* $||A_{\cdot,j}|| = 1$;
9. $\quad \tau^{(i)} = \tau^{(i-1)} - d\tau$;
10. **end**
11. **return** $S^{(I_{max})}, A^{(I_{max})}$

Preliminary results

The accuracy of the mixing matrix **A** is critical to extract the sources. Therefore, we choose a criterion of estimated **A** such as $c_A = \log\frac{||A_{est}^\dagger A_{ref} - I_n||}{n^2}$. Our tests are effectuated on prior sparsity of direct domain with mask=50%. The figure below shows that in different cases of sources numbers(2,3,4,5 from left to right), the criteria of estimated **A** in terms of band numbers. We can conclude that with band numbers increased, generally the mixing matrix is better evaluated. In addition, with source number increased, the accuracy of mixing matrix is degraded.



Conclusions

- The HSS-ForWaRD is an on-going work. The next steps will study the sources which are not sparse in direct domain but in wavelet domain.
- We will also treat more complicated cases not only incomplete data but also the instrumental PSF issue. In addition, more complicated noise will be taken into account.

References

- Neelamani, R.N.; Hyeokho Choi; Baraniuk, R., ForWaRD: Fourier-wavelet regularized deconvolution for ill-conditioned systems,in Signal Processing, IEEE Transactions on , vol.52, no.2, pp.418-433, Feb. 2004