Fourier dimensionality reduction for fast radio transients

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***Abstract*—In the context of next-generation radio interferometers, we are facing a big challenge of how to economically process data. The classical dimensionality reduction technique, averaging visibilities on time, may dilute fast radio transients (FRT). We propose a robust fast approximate SVD-based dimensionality reduction method for FRT imaging. For each time slice of FRT imaging, our dimensionality reduction defines a linear embedding operator to reduce the space spanned by the left singular vectors of the measurement operator and this operator can be fast obtained via a weighted fft on adjoint measurement operator instead of expensive SVD. The preliminary results showcase that the proposed dimensionality reduction can simultaneously reduce the data significantly and recover FRT correctly, while the averaging technique causes the FRT dilution problem.**

I. Introduction

Fast radio transients (FRT) detection is an active field of research in radio astronomy and modern radio interferometers have seen the development of the FRT detection pipeline in recent years. Unlike steady sources imaging, the FRT imaging stressing temporal information recovery suffers from the sensitivity of the integration time of the instrument. On the one hand, short integration time leads to poor uv coverage and FRT detection is ambiguous. On the other hand, long integration time can ensure good uv coverage, but FRT are likely diluted temporally. In addition, tremendous data are produced in the modern radio interferometers, which addresses a computational challenge for imaging.

In this context, we present herein a dimensionality reduction method to tackle big data challenge, which can be incorporated with optimization-based FRT imaging methods ([1]) as an upstream data pre-processing module.

II. Method

The FRT imaging problem can be summarized as $\mathbf{y}_t = \mathbf{\Phi}_t \mathbf{x}_t + \mathbf{n}_t$, where $\mathbf{y}_t \in \mathbb{C}^M$ denotes continuous visibilities at time frame t, corrupted by additive i.i.d. Gaussian noise $n \in \mathbb{C}^M$ and $\mathbf{\Phi}_t \in \mathbb{C}^{M \times N}$ is the measurement operator to measure the sky $\mathbf{x}_t \in \mathbb{R}^N$ ($M \gg N$) at time frame t. By applying an embedding linear operator $\mathbf{R}_t$ (at t), the reduced imaging problem is written as $\mathbf{R}_t \mathbf{y}_t = \mathbf{R}_t \mathbf{\Phi}_t \mathbf{x}_t + \mathbf{R}_t \mathbf{n}_t$.

As the measurement operator is time varying in FRT imaging, we only reduce the spatial dimensionality for each time frame. The optimal dimensionality reduction $\mathbf{R}_t$ is a projection to the left singular vectors $\mathbf{U}_t$ of the measurement operator $\mathbf{\Phi}_t$ by selecting the most significant singular values. Thus, thanks to the SVD decomposition, $\mathbf{R}_t = \mathbf{U}_{t_{th}}^{\dagger} = \mathbf{\Sigma}_{t_{th}}^{-1} \mathbf{V}_{t_{th}}^{\dagger} \mathbf{\Phi}_{t_{th}}^{\dagger}$, where $\mathbf{U}_{t_{th}}^{\dagger}$ is singular vectors corresponding to singular values after a thresholding of th. However, due to the computational demanding SVD, a fast approximation ([2]) is proposed such that $\mathbf{V}_t{}^{\dagger} \approx \mathbf{F}$ and $\mathbf{\Sigma}_t{}^2 \approx \mathrm{Diag}(\mathbf{F}\mathbf{\Phi}_t^{\dagger}\mathbf{\Phi}_t\mathbf{F}^{\dagger})$ owing to the fact that the matrix $\mathbf{F}\mathbf{\Phi}_t^{\dagger}\mathbf{\Phi}_t\mathbf{F}^{\dagger}$ is diagonal dominant. Therefore, the final reduction operator at time frame t is given by $\mathbf{R}_t = \mathbf{\Sigma}_t{}^{-1}\mathbf{S}_{th}\mathbf{F}\mathbf{\Phi}_t{}^{\dagger}$, where the selection matrix $\mathbf{S}_{th}$ selects singular values larger than the given threshold th. After this reduction, the data size is reduced to $M_{th} < N$. Then we can leverage the convex optimization-based algorithm ([1]) for FRT detection.

III. Experiments

In the radio astronomy community, a widely used technique to reduce the data dimensionality is to average visibilities on time slices (*averaging* hereafter), allowing a better SNR and a smaller data set. Nevertheless this technique may dilute temporal information. In order to compare our dimensionality reduction with this averaging technique, we create the sky model cube of size $32 \times 32 \times 256$ with 256 time frames each of them represents 45 seconds and 5 FRT whose "lifespan" are between 1.5min (2 snapshots) and 4.5min (6 snapshots). The realistic uv-coverage is simulated via MeqTree tool by using SKA 197 antennas configuration. The additive noise is directly injected on the visibilities with SNR=-30dB (SNR $= 20\log_{10}(||\mathbf{y}_0||_2/||\mathbf{n}||_2)$). The results of three scenarios in fig. 1 are presented: recovery with all data, with reduced data by proposed reduction and with reduced data by averaging. We can observe that our reduction method does not degrade reconstruction quality, achieving almost the same result of all data. However, the averaging showcases the problem of dilution where the FRT can not be detected at all.

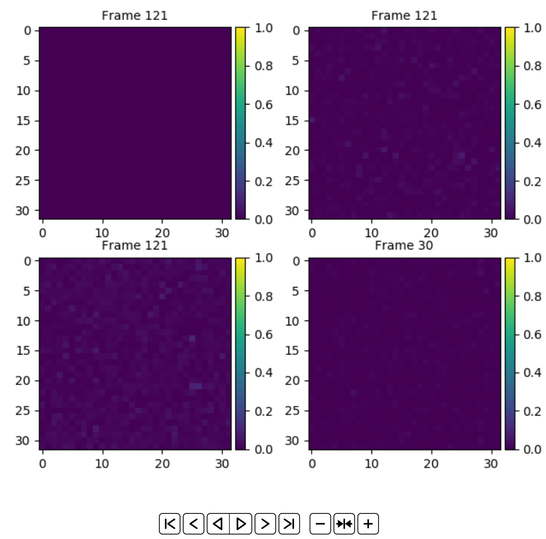

Fig. 1. Numerical experiments on simulated FRT (only present frames from 121 to 128). Left-to-right then top-to-bottom: sky model, reconstruction with all data, reconstruction with 25% data by using proposed reduction method and reconstruction with 25% data by using averaging.

References

[1] M. Jiang, J. N. Girard, J. . Starck, S. Corbel, and C. Tasse, "Compressed sensing and radio interferometry," in *2015 23rd European Signal Processing Conference (EUSIPCO)*, Aug 2015, pp. 1646–1650.

[2] S. Vijay Kartik, R. E. Carrillo, J.-P. Thiran, and Y. Wiaux, "A fourier dimensionality reduction model for big data interferometric imaging," *Monthly Notices of the Royal Astronomical Society*, vol. 468, no. 2, pp. 2382–2400, 2017.